Sample Complexity of Differential Privacy

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Privacy Tools for Sharing Research Data

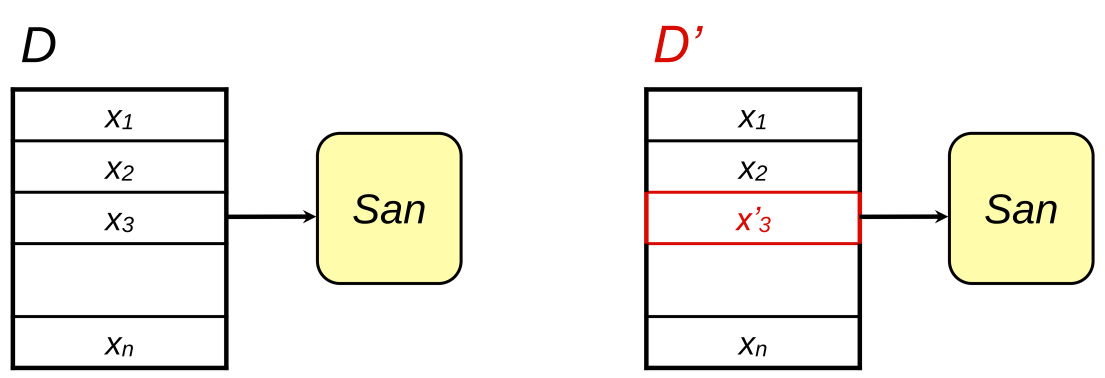
A National Science Foundation Secure and Trustworthy Cyberspace Project

MAIN QUESTION

How many **data samples** do we need to achieve both **differential privacy** and **statistical accuracy**?

i.e. How big a study do we need to conduct to answer our questions and preserve privacy?

DIFFERENTIAL PRIVACY



D and D' are neighbors if they differ only on one user's data

An algorithm *San* is (ε,δ)-differentially private if for all neighbors D, D' and every $S \subseteq \mathrm{Range}(San)$,

$$\Pr[San(D) \in S] \leq e^{\varepsilon} \Pr[San(D') \in S] + \delta$$

Think of $\varepsilon = \Theta(1)$ and $\delta = o(1/n)$

ACCURACY FOR COUNTING QUERIES

Counting queries: What fraction of rows in a database satisfy property q?

e.g. $q(x)$ = LikesBread AND LikesToast

LikesBread?	LikesButter?	LikesToast?	LikesJam?	
0	0	1	1	$q(x_1)=0$
1	1	1	1	$q(x_2)=1$
1	0	1	0	$q(x_3)=1$

d (=4) attributes per record

$q(D)=2/3$

Answers a_q are α-accurate if $|a_q - q(D)| < \alpha$ for every $q \in Q$

SAMPLE COMPLEXITY UPPER BOUNDS

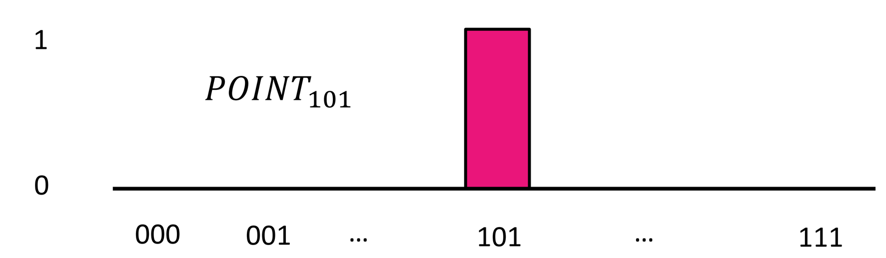
For general queries Q,

$$O(\sqrt{d} \log |Q| / \alpha^2)$$

samples suffice [HR10], using the analysis in [GRU12]

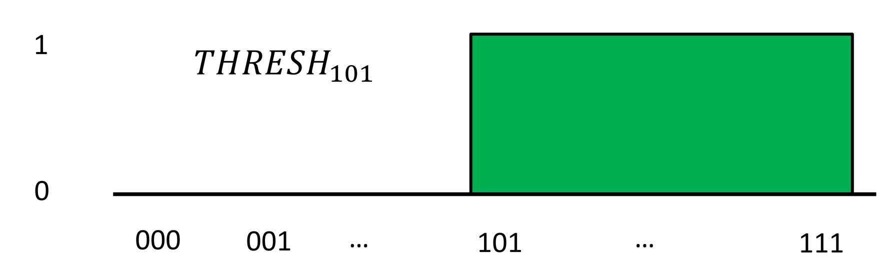
But for certain Q, the sample complexity can be much lower:

Point queries: $POINT_y(x) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$



$\log|Q| = d$, but just $O(1/\alpha)$ samples suffice

Threshold queries: $THRESH_y(x) = \begin{cases} 1 & \text{if } x \geq y \\ 0 & \text{otherwise} \end{cases}$



Again, $\log|Q| = d$, but $<< d/\alpha^{2.5}$ samples suffice.[BNS13]

- Extend to upper bounds on the sample complexity of differentially private *PAC learning.*
- Sample complexity is much smaller than what is needed for pure (i.e. δ = 0) privacy.
- Relevant quantity seems to be the VC-Dimension of Q

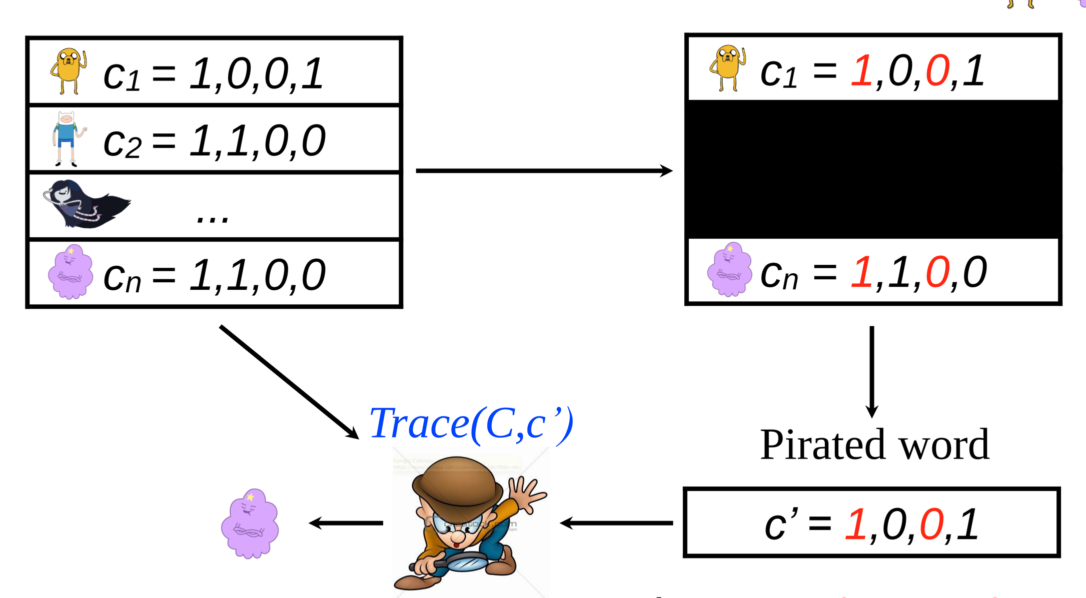
SAMPLE COMPLEXITY LOWER BOUNDS

Our contributions [BUV13]

- To answer *arbitrary* queries, $\Omega(\sqrt{d} \log |Q| / \alpha^2)$ samples are necessary (nearly tight)
- If α is a constant, this lower bound still holds for *conjunction* queries

Tool 1: Fingerprinting Codes

Coalition of users $S \subseteq [n]$



c' must be feasible for S: for every $j \in [d]$, exists $i \in S$ s.t. $c'_j = c_{ij}$

- Sensitive database = traceable codebook
- Traceability is the "opposite" of privacy
- Yields a lower bound of $\Omega(\sqrt{d})$ for estimating the mean of each column

Tool 2: Reconstruction Attacks [DN03]

						t		$\hat{t}$
0	1	1	1	1	1	0		0
1	0	1	1	1	1	1		0
1	1	0	1	1	1	1		1
1	1	1	0	1	1	1		1
1	1	1	1	0	1	0		0
1	1	1	1	1	0	0		0

Last column of "sensitive bits" First k columns are fixed, public, "names"

Goal is to reconstruct most of the sensitive bits from accurate answers to ($k+1$)-way conjunctions

Approximate answers to ($k+1$)-way conjunctions on D

COMPOSITION OF LOWER BOUNDS

0	1	1	1	0	1	0	1	
0	1	1	1	1	1	0	0	$D_1 \in (\{0,1\}^d)^m$
0	1	1	1	0	1	1	1	
1	0	1	1	0	1	0	0	
1	0	1	1	1	0	1	0	$D_2 \in (\{0,1\}^d)^m$
1	0	1	1	0	1	1	1	
⋮								
1	1	1	0	1	1	0	1	
1	1	1	0	1	0	0	0	
1	1	1	0	0	1	1	1	$D_k \in (\{0,1\}^d)^m$

Random stack of "sensitive databases"
First k columns are public, fixed "names" for each D_i

Goal is to answer (most) 1-way conj's on at least one D_i ⇒ privacy breach

($k+1$)-way conj's compute "subset sums of 1-way conj's"

REFERENCES

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[HR10] Moritz Hardt and Guy Rothblum. A multiplicative weights mechanism for privacy-preserving data analysis. In FOCS, 2010.

ACKNOWELEDGMENTS

*Based mostly on joint work with Jonathan Ullman and Salil Vadhan [BUV13]

Most slides are courtesy of Jonathan Ullman

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